Surface Areas and Volumes

Assertion & Reason Type Questions

Directions: In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct choice as:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c. Assertion (A) is true but Reason (R) is false.

d. Assertion (A) is false but Reason (R) is true.

Q1. Assertion (A): If the ratio of the radii of two right circular cones of the same height is 1:3, then the ratio of their curved surface area when the height of each cone is 3 times the radius of the smaller cone, is $\sqrt{5}$: 9.

Reason (R): The ratio of curved surface area of two different cones cannot be in the same ratio of their radii.

Answer : (c) Assertion (A): Given,

$$\frac{r}{R} = \frac{1}{3} \Rightarrow R = 3r$$

Let the radius of the first (smaller) cone be 'x' and the height of the cone will be 3x.

$$\therefore \quad l = \sqrt{h^2 + r^2} = \sqrt{(3x)^2 + x^2}$$

$$= \sqrt{10x^2} = \sqrt{10}x$$

$\therefore$ CSA of first cone $= \pi rl$ sq. units

$$= \pi x \sqrt{10}x = \pi x^2 \sqrt{10}$$

The radius and the height of the second (bigger) cone is 3x

$$\therefore \quad l = \sqrt{(3x)^2 + (3x)^2} = \sqrt{9x^2 + 9x^2}$$

$$= \sqrt{18x^2} = 3\sqrt{2}x$$

CSA of second cone $= \pi \times 3x \times 3\sqrt{2}x$ sq. units

$$= 9\pi\sqrt{2}x^2$$

$\therefore$ Ratio of the curved surface area

$$= \pi x^2\sqrt{10} : 9\pi x^2\sqrt{2}$$

$$= \sqrt{10} : 9\sqrt{2} = \sqrt{5} : 9$$

$\therefore$ Assertion (A) is true.

Reason (R): It is not always true to say that the ratio of curved surface area of two different cones cannot be in the same ratio of their radii.

Hence, Assertion (A) is true, but Reason (R) is false.

Q2. Assertion (A): If the volumes of two spheres in the ratio 125 : 8, then their surface area are in the ratio 25 : 4.

Reason (R): If R is the radius of a sphere, then volume and surface area of sphere are $\frac{4}{3}\pi r^3$ cubic units and $4\pi R^2$ sq. units.

Answer : (a) Assertion (A): Given, volume of two spheres are in the ratio

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi R_1^3}{\frac{4}{3}\pi R_2^3}$$

$$\Rightarrow \quad \frac{125}{8} = \left(\frac{R_1}{R_2}\right)^3$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{5}{2}$$

$\therefore$ The ratio of the surface areas of two spheres are in the ratio

$$\frac{S_1}{S_2} = \frac{4\pi R_1^2}{4\pi R_2^2}$$

$$= \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Q3. Assertion (A): A shotput is a metallic sphere of radius 4 cm. If the density of the metal is 10 gm per cm^3, then the mass of the shotput is 2 kg.

Reason (R): Volume of sphere with radius r is $\frac{4}{3}\pi r^3$.

Answer : (d) Assertion (A): Given, radius of metallic sphere is r = 4 cm

$$\therefore \text{ Volume of metallic sphere } = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (4)^3$$

$$= 268.19 \text{ cm}^3$$

$\because$ Density of metal = 10 gm/cm^3

$\therefore$ Mass of shotput = Density $\times$ Volume

$$= 10 \times 268.19$$

$$= 2681.9 \text{ gm}$$

$$= \frac{2681.9}{1000} \text{ kg} \qquad \left[\because 1 \text{ gm} = \frac{1}{1000} \text{kg}\right]$$

$$= 2.68 \text{ kg} \neq 2 \text{ kg}$$

So, Assertion (A) is false.

Reason (R): It is true.

Hence, Assertion (A) is false but Reason (R) is true.

Q4. Assertion (A): If diameter of a sphere is 20 cm and it is reduced by 10% then the change in its volume will be 26.1%.

Reason (R): Change in volume

Answer : (d) Assertion (A): Given, diameter of sphere be 20 cm, original diameter,

D_1 = 20 cm

$$\therefore \text{ Original radius of sphere, } R_1 = \frac{20}{2} = 10 \text{ cm}$$

$$\text{and original volume of sphere, } V_1 = \frac{4}{3}\pi R_1^3$$

$$= \frac{4}{3}\pi \times 10 \times 10 \times 10 \text{ cm}^3$$

$$= \frac{4}{3}\pi \times 1000 \text{ cm}^3$$

New diameter, $D_2 = (100\% - 10\%)$ of 20 cm

$$= \frac{90}{100} \times 20 = 18 \text{ cm}$$

New radius, $R_2 = \frac{18}{2} = 9$ cm

$\therefore$ New volume of sphere, $V_2 = \frac{4}{3}\pi R_2^3$

$$= \frac{4}{3}\pi \times 9 \times 9 \times 9 \text{ cm}^3$$

$$= \frac{4}{3}\pi \times 729 \text{ cm}^3$$

Now, change in volume $= \frac{V_1 - V_2}{V_1} \times 100$

$$= \frac{\frac{4\pi}{3} \times 1000 - \frac{4\pi}{3} \times 729}{\frac{4\pi}{3} \times 1000} \times 100$$

$$= \frac{4\pi(1000 - 729) \times 3 \times 100}{3 \times 1000 \times 4\pi}$$

$$= \frac{271}{1000} \times 100 = 27.1\ \%$$

Assertion (A) is false.

Reason (R): It is true statement.

Hence, Assertion (A) is false but Reason (R) is true.